INFLUENCE OF CONDUCTOR SHIELDS ON THE Q-FACTORS OF A TE_0 DIELECTRIC RESONATOR

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ABSTRACT

Based on the mode-matching method useful for computing the accurate resonant frequencies, two approaches due to the complex frequency and to the perturbation theory are described to accurately compute Q-factors of TE_0 dielectric rod resonators placed between two parallel conductor plates and in a conductor cavity. Influence of the conductor shields on the Q-factors is discussed from the computed results.

INTRODUCTION

Dielectric resonators widely used in microwave circuits are placed in conductor shields to prevent radiation loss. The analyses of Q-factors for such shielded resonators have been treated by several authors [1]-[6]. As the first approach, we commonly use the definition Q=ω(the energy stored)/(the average power dissipated), where ω is the resonant angular frequency. However, its analysis for such resonators is quite involved since the exact field expressions are very complicated [1], and therefore simplifying approximations are considered [2], [3]. As the second approach, a conception of the complex frequency is introduced into the characteristic equations to simultaineously determine the resonant frequencies and Q-factors. This method is useful for computing the Q-factors due to dielectric and radiation losses, Q_d and Q_{rad} [4],[5]. A technique of computing one due to the conductor loss Q_c in a similar manner has also been presented by Maj and Modelski [5]. However their procedure, which considers a conductor layer in the conductor wall, appears to be rather complicated. The third approach based on the perturbation of cavity walls has been presented by Kajfez [6]. This is the most available method for the Q_c computation of the TE_0 mode, since it requires the frequency computation only. However, a similar analysis for Q_d have not been treated so far.

In this paper, based on the mode-matching method useful for computing the accurate resonant frequencies [7],[8], two approaches due to the complex frequency and to the perturbation theory are described to accurately compute the Q-factors of the TE_0 modes for resonator structures shown in Fig. 1. The extension of Kajfez's method to the Q_d computation can be realized by means of the cavity-material perturbations. These considerations allow us to separately estimate the influence of the conductor shields on the Q_c and Q_d values. Validity of the theories is verified by experiments.

CHARACTERISTIC EQUATIONS

Consider three types of shielded dielectric resonators shown in Fig. 1. A dielectric rod of relative permittivity ε_r and diameter D is placed between two parallel conductor plates as in Fig. 1(a) or (b), or in the center of a conductor cavity of diameter d and length 2h as in Fig. 1(c). They are called parallel-plates-image, parallel-plates-open, and cavity-open types, respectively. The conductor and dielectric are supposed to be lossless first. From the structural symmetry in Fig. 1(b) the resonant modes can be classified into those for which T-plane (rθ-plane at z=0) is an electric wall and the others for which it is a magnetic wall. The former case also corresponds to Fig. 1(a). An analysis for the TE_0 mode can be performed by the mode expansion method. As the result we obtain the following characteristic equation [7]:

$$\det H = 0 \tag{1}$$

where the matrix element h_{qp} (q, p=1, 2,..., N) is given by

$$h_{qp} = \frac{\left[\frac{J_1(u_p)}{u_p J_0(u_p)} - \frac{H_1(v_q)}{v_q H_0(v_q)}\right]\cdot F_{qp}}{[X_p^2 - Z_q^2][(Y_p/M)^2 - (Z_q/L)^2]} \tag{2}$$

$$F_{qp} = \left\{ \begin{matrix} X_p \cot X_p - Z_q \cot Z_q \\ X_p \tan X_p - Z_q \tan Z_q \end{matrix} \right\} \tag{3}$$

$$u_p = \sqrt{\left(\frac{\omega R}{c}\right)^2 \varepsilon_r - \left(\frac{R}{L}X_p\right)^2}, \quad v_q = \sqrt{\left(\frac{\omega R}{c}\right)^2 - \left(\frac{R}{L}Z_q\right)^2} \tag{4}$$

$$Z_q = \left\{ q\pi\frac{L}{h},\ (2q-1)\pi\frac{L}{2h} \right\} \tag{5}$$

In the above the upper and lower (or the first and second) expressions in { } correspond to the electric and magnetic T-plane modes, respectively. A time factor $e^{j\omega t}$ is tracitly assumed. Also c is the light velocity in a vacuum, $J_n(x)$ the Bessel function of the first kind, and $H_n(x)$ the Hankel function of the second kind. Furthermore (X_p, Y_p) is given as the p'th solution of the following simultaneous equations:

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$$\left.\begin{array}{l}\{-X\cot X,\ X\tan X\}=\frac{L}{M}Y\cot Y\\ (X/L)^2-(Y/M)^2=(\omega/c)^2(\varepsilon_r-1)\end{array}\right\} \quad (6)$$

When $\{h, 2h\} > \lambda_0/2$, where λ_0 is the resonant frequency, the resonant modes are in a leaky state [8]: a part of energy leaks away from the resonator in the radial direction. On the other hand, when $\{h, 2h\} \leq \lambda_0/2$, the resonant modes are in a trapped state: the energy is trapped in and near the rod. In this case, it follows that $v_q=-jv'_q$ (v'_q is real number) for any q value, and then the term containing the Hankel functions in (2) is rewritten as

$$\frac{H_1(v_q)}{v_q H_0(v_q)} \rightarrow -\frac{K_1(v'_q)}{v'_q K_0(v'_q)} \quad (7)$$

where $K_n(x)$ is the modified Bessel function of the second kind. Particularly, the case of $\{h, 2h\}=\lambda_0/2$ represents a cutoff of the trapped state.

Furthermore, for the cavity-open type resonator in Fig. 1(c) the following exchange only is needed in (2):

$$\frac{H_1(v_q)}{v_q H_0(v_q)} \rightarrow \frac{I_1(v'_q)K_1(v'_qS)-I_1(v'_qS)K_1(v'_q)}{v'_q[I_0(v'_q)K_1(v'_qS)+I_1(v'_qS)K_0(v'_q)]} \quad (8)$$

where $S=a/R$ and $I_n(x)$ is the modified Bessel function of the first kind.

ANALYSIS BY COMPLEX FREQUENCY TECHNIQUE

As the first approach we introduce the complex angular frequency

$$\dot{\omega}=\omega_1+j\omega_2; \quad f_1=\omega_1/2\pi, \quad Q_f=\omega_1/2\omega_2, \quad (9)$$

and the complex relative permittivity

$$\dot{\varepsilon}_r=\varepsilon_r(1-j\tan\delta) \quad (10)$$

into (1), where f_1 and Q_f are the resonant frequency and Q-factor for a damped-free oscillation, respectively, and $\tan\delta$ is the loss tangent of dielectric.

ANALYSIS BY PERTURBATION TECHNIQUE

Following Kajfez's method, we can compute the Q_c values from

$$\frac{1}{Q_c}=\frac{1}{Q_{cu}}+\frac{1}{Q_{cl}}, \quad \frac{1}{Q_c}=\frac{2}{Q_{cu}}, \quad \frac{1}{Q_c}=\frac{2}{Q_{cu}}+\frac{1}{Q_{cy}}, \quad (11)$$

for Fig. 1(a), (b), and (c), respectively, where

$$Q_{cu}=\frac{f_0}{(-\Delta f_0/\Delta M)\delta_c}, \quad Q_{cl}=\frac{f_0}{(-\Delta f_0/\Delta L)\delta_c}, \quad Q_{cy}=\frac{f_0}{(-\Delta f_0/\Delta d)\delta_c}, \quad (12)$$

and Q_{cu}, Q_{cl}, and Q_{cy} are ones due to the conductor losses of the upper and lower plates and of the cylinder, respectively. The frequency shift Δf_0 due to the cavity-wall perturbation Δx, where x represents M, L, or d, can be numerically and accurately computed from (1). Also $\delta_c=(\pi f_0\mu\sigma)^{-1/2}$ is the skin depth of the conductor.

Similarly Q_d can be derived by means of the cavity-material perturbations. At first, from the definition of Q_d we obtain

$$Q_d=\frac{1}{\tan\delta}\,\frac{W_d+W_a}{W_d} \quad (13)$$

where W_d and W_a are the electric energy stored in the dielectric and air, respectively. Then we obtain

$$\frac{\Delta f_0}{f_0}=-\frac{\Delta\varepsilon_r}{2\varepsilon_r}\,\frac{W_d}{W_d+W_a} \quad (14)$$

from the frequency shift Δf_0 due to the material perturbation $\Delta\varepsilon_r$ in a cavity including dielectrics [9]. Hence from (13) and (14) the following formula can be obtained:

$$Q_d=\frac{1}{\tan\delta}\,\frac{f_o}{(-\Delta f_0/\Delta\varepsilon_r)2\varepsilon_r} \quad (15)$$

where the value of $\Delta f_0/\Delta\varepsilon_r$ also can be computed from (1). Thus, using (11) and (15), we can obtain the unloaded Q, Q_u from

$$\frac{1}{Q_u}=\frac{1}{Q_c}+\frac{1}{Q_d} \quad (16)$$

This method is valid only without radiation.

COMPUTATION AND EXPERIMENT

The computation and experiment were performed using a $(Zr.Sn)TiO_4$ ceramic rod (MURATA MFG.CO., LTD.) with $\varepsilon_r=37.43$ and $\tan\delta=(0.205+0.170f_0[\mathrm{GHz}])\times10^{-4}$ and two copper plates with $\bar{\sigma}=\sigma/\sigma_0=0.92$, where $\sigma_0=58\times10^6$ S/m. These values were measured by a dielectric rod resonator method [10].

At first, for the $TE_{01(1+\delta)/2}$ mode of the parallel-plate-image type resonator in Fig. 1(a) and for the $TE_{01\delta}$ mode of the parallel-plate-open type one in Fig. 1(b), the complex frequency versus the distance M was computed using (1)-(6) with (9) and (10). The respective results are shown in Figs. 2 and 3 by solid lines. Broken lines in the figures show the cutoffs. The left-hand side of the cutoff is the trapped state region, while the right-hand side is the leaky state region.

In addition the measured values of the resonant frequency f_0 and the unloaded Q, Q_u by the swept-frequency method are indicated by dots in the figures. In both cases, the theoretical f_1 curves agree very well with the measured f_0 values. The theoretical Q_f curves in the trapped state, which actually means Q_d, are greater than the measured Q_u values since the conductor loss is not considered in the analysis, while the Q_f curves in the leaky state, which consist of Q_d and Q_{rad}, agree well to the measured Q_u values since the radiation loss is predominant.

In the following, for the same structures the Q_c, Q_d, and Q_u values in the trapped state were computed using (1)-(7), (11), (12), (15), and (16). The respective results are shown in Figs. 4 and 5, where the computed f_0 curves are not shown since they are identical to the f_1 curves given in Figs. 2 and 3, respectively. In both structures, the Q_d values computed from (15) agree to within 0.05 percent with the computed Q_f curves in the trapped states in Figs. 2 and 3. Moreover, the computed Q_u curves agree very well with the measured Q_u values which are reproductions of those in Figs. 2 and 3. Thus validity of two methods described above was verified. It should be noted that the Q_c values increase rapidly with increasing M.

POSSIBILITY OF HIGH-Q DIELECTRIC RESONATORS

Finally, for the $TE_{01\delta}$ mode of the cavity-open type resonator in Fig. 1(c) with the optimum dimensions to obtain the best separation of higher-order modes [8], the f_0 and Q_u values were computed using (1)-(6), (8), (11), (12), (15), and (16). When ε_r= 37.5 and D=10 mm, the optimum values are 2L=4.19 mm, M=5.26 mm, and d=27.0 mm, and then we obtain f_0 =5.37 GHz and $Q_d \tan\delta$=1.026. For $\bar{\sigma}$=1.0 (copper) we obtain $Q_{cu}=3.40\times10^5$, $Q_{cy}=5.54\times10^5$, and therefore $Q_c=1.30\times10^5$. Thus we obtain Q_u=9,520 when $\tan\delta = 1\times10^{-4}$ and also Q_u=57,000 when $\tan\delta=1\times10^{-5}$. For a TE_{011} empty cavity, on the other hand, the theoretical maximum Q_u value attained when d=2h, is 41,000 at f_0=5.4 GHz. As a result, if low-loss materials with $\tan\delta$ of nearly 10^{-5} are developed, shielded dielectric resonators will realize the Q_u values higher than those of conductor cavities.

CONCLUSION

It is concluded that two approaches presented are effective for the separate and accurate estimation of the Q-factors due to the dielectric, conductor, and radiation losses for the TE_0 modes of the shielded dielectric resonators. The computed results show that the Q value due to the conductor loss increases rapidly as the conductor is moved gradually away from the dielectric. As a result, a possibility of realizing high-Q dielectric resonators in the microwave region was suggested. In addition, a practical application of such resonators in the millimeter wave region also can be expected as suggested by Dydyk [11].

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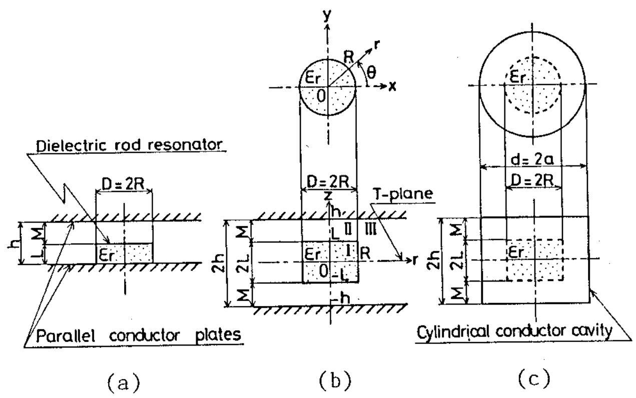

Fig. 1. Configurations of shielded dielectric rod resonators of three types; (a) parallel-plate-image type, (b) parallel-plate-open type, and (c) cavity-open type.

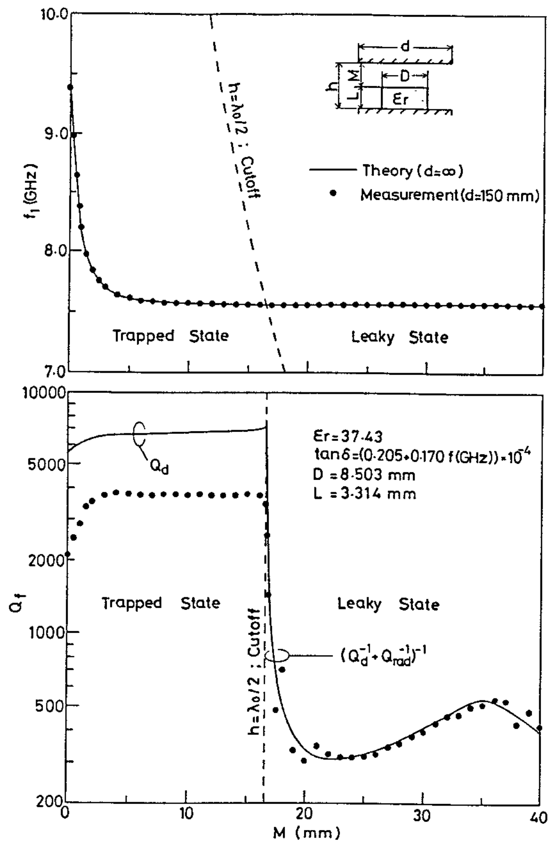

Fig. 2. Computed results of complex frequency (f_1, Q_f) and measured values (f_0, Q_u) for the $TE_{01(1+\delta)/2}$ mode of the parallel-plate-image type resonator.

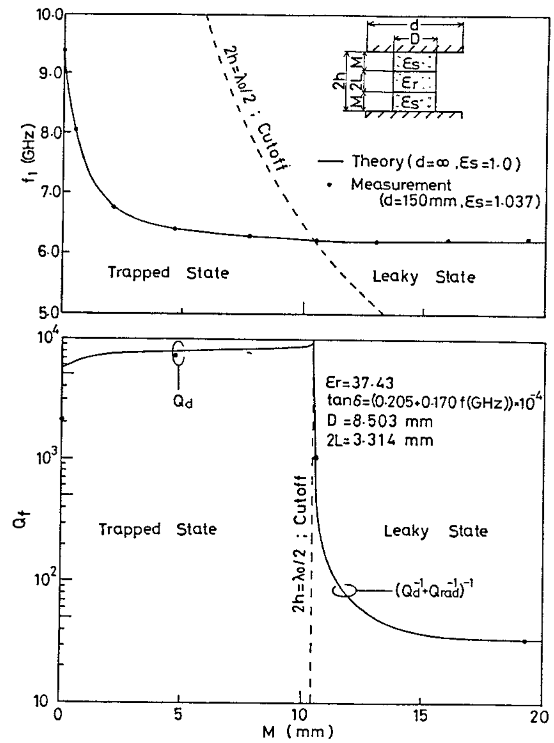

Fig. 3. Computed results of complex frequency (f_1, Q_f) and measured values (f_0, Q_u) for the $TE_{01\delta}$ mode of the parallel-plate-open type resonator.

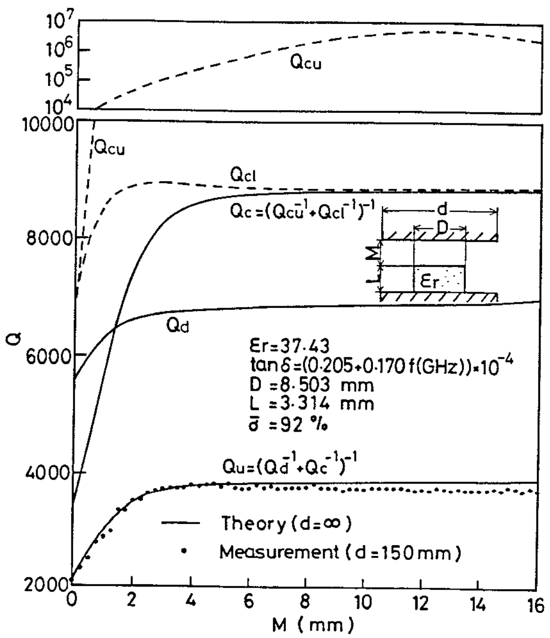

Fig. 4. Computed Q values due to the perturbation theory and measured Q_u values for the trapped state $TE_{01(1+\delta)/2}$ mode of the parallel-plate-image type resonator.

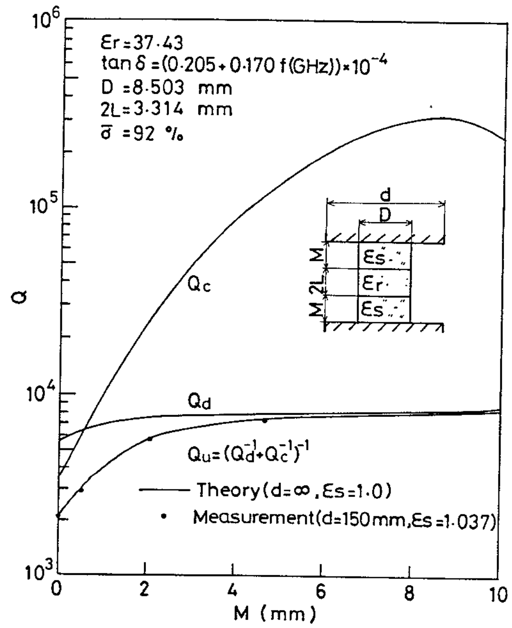

Fig. 5. Computed Q values due to the perturbation theory and measured Q_u values for the trapped state $TE_{01\delta}$ mode of the parallel-plate-open type resonator.